COMPARATIVE ANALYSIS OF STRESS AND STRAIN FOR FUNCTIONALLY GRADED RECTANGULAR CANTILEVER BEAM

By-Maham Zia

Email id-mahamziame0019@gmail.com

1. INTRODUCTION

Functionally graded materials (FGMs) are composites in which the volume fraction, sizes, and shapes of material constituents can be varied to get desired smooth spatial variations of macroscopic properties such as the elastic modulus, the mass density, the heat conductivity, etc. to optimize their performance. The FGMs abound in nature, e.g., human teeth, bamboo stick, sea shell. Engineered FGMs include ceramic–metal and fiber-reinforced polymeric composites, concrete, and rubberlike materials. Vulcanized rubber components typically exhibit a spatial variation of mechanical properties caused either by thermal gradients during their fabrication or chemical changes induced due to interaction with the environment during their service.

Rubberlike materials are widely used in aerospace, automotive, and biomedical fields. They are usually regarded as incompressible, can thus undergo only isochoric or volume preserving deformations, and their constitutive relation involves hydrostatic pressure that cannot be determined from the deformation field but is to be found as a part of the solution of the boundary-value problem (BVP). The BVPs for functionally graded incompressible materials (FGIMs) are challenging since the governing differential equations have variable coefficients and it is difficult to find their exact solutions. In general, the solution of a BVP for a structure composed of an FGIM cannot be obtained from that of the corresponding problem for a compressible material by setting Poisson's ratio equal to 0.5. Furthermore, the solution for a plane stress problem cannot be obtained from that for a plane strain problem by modifying Young's modulus E and Poisson's ratio ν. We briefly review below the literature on FG rotating disks and other works for FGIMs.

2. WHAT IS FGM?

Functionally Graded Materials, are revolutionary material, belongs to a class of advanced materials, composition and structure gradually change over volume therefore changing the properties of the material in order to perform a certain function(s). Functionally Graded Material happen in nature as bones, teeth and so forth, nature planned this material to meet their expected administration prerequisites. This thought is imitated from nature to tackle designing issue the same way simulated neural system is utilized to copy human mind. Practically evaluated material, takes out the sharp interfaces existing in composite material which is the place disappointment is started. It replaces this sharp interface with an inclination interface which creates smooth move from one material to the following. One of qualities of FGM is the capacity to tailor a material for particular application. FGMs were initially designed as thermal barrier materials for aerospace structural applications and fusion reactors. They are now developed for general use as structural components in extremely high-temperature environments. FGMs are now recognized as important composite materials throughout the world.

Utilization of FGMs appears to be one of the most efficient and effective materials in achieving sustainable development in Industries.

FGMs are those revolutionary materials, composition or microstructure is varied so that a certain variation of the local material properties is achieved. FGM is also defined as, those in which the volume fraction of two or more materials are achieved continuously as a function of position along certain direction of the structure to achieve a required function. The basic configurations of composites and FGMs and differences between constituents are shown in Figure 1.2. The main role of the metal constituent in FGM is to provide the structural support, while the other constituent is to provide heat shielding or thermal barrier when subjected to high temperature environments. The material property variation makes it possible to accommodate the function to appropriate the needs of different applications.

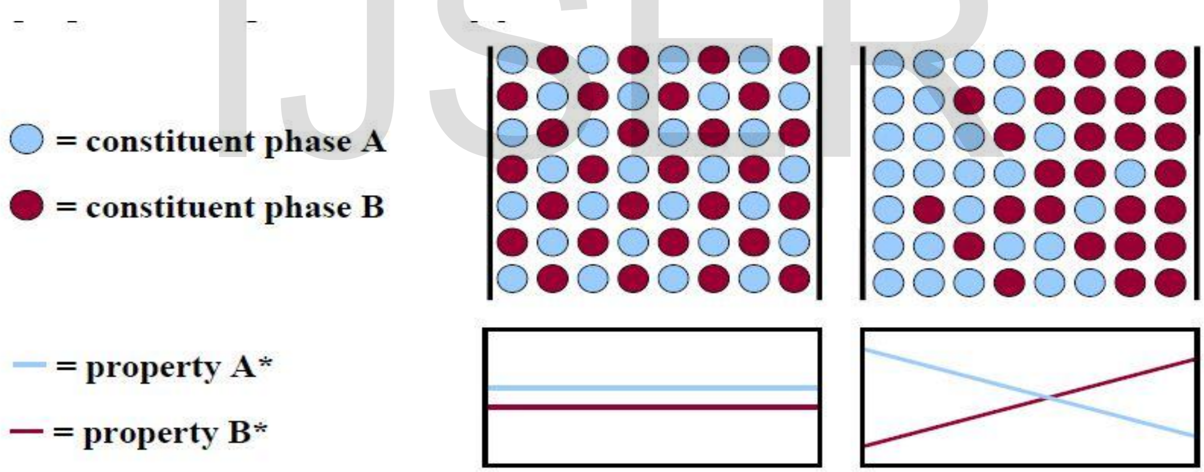

Figure1.1 configuration of composites and FGM

Composite materials will fail under extreme working conditions through a process called delamination (separation of fibers from the matrix). This can happen for example, in high temperature application where two metals with different coefficient of expansion are used. To solve this problem, researchers in

Japan in the mid- 1980s,confronted with this challenge in an hypersonic space plane project requiring a thermal barrier (with outside temperature of 2000K and inside temperature of 1000K across less than 10 mm thickness), came up with a novel material called Functionally Graded Material (FGM).

As many thin wall members, plates , i.e., may be rectangular or circular used in reactor vessels, turbines and other machine parts are susceptible to failure from large amplitude of deflection or excessive stresses induced by thermal or thermo mechanical loading. Thus FGMs are primarily used in structures subjected to extreme temperature environment or where high temperature gradients are encountered. Mainly they are manufactured from isotropic components such as metals and ceramics, since role of metal portion is acts as structure support while ceramics provides thermal with severe thermal gradients. In such conditions ceramic provides heat and corrosion resistance, while the metal provides the strength and toughness.

FGM permits the certain unrivaled and various properties with no mechanically frail interface. This new idea of materials relies on material science and mechanics because of joining of the material and auxiliary thought into the last outline of basic component. Moreover, steady change of properties can be custom-made to diverse applications and administration situations. The rundown is perpetual and more application is springing up as the preparing innovation, expense of creation and properties of FGM moved forward.

3.MANUFACTURING PROCESSS OF FGMs

Functionally Graded Materials are ordinarily in the type of surface coatings, there are an extensive variety of surface statement procedures to browse contingent upon the administration necessity from the procedure.

A. Vapor Deposition Technique

B. Powder Metallurgy (PM)

C. Centrifugal Method

D. Solid Freeform (SFF) Fabrication Method

There are different types of vapor deposition techniques, they include: sputter deposition, Chemical Vapor Deposition (CVD) and Physical Vapor Deposition (PVD). These vapor deposition methods are used to deposit functionally graded surface coatings and they give excellent microstructure, but they can only be used for depositing thin surface coating. They are energy intensive and produce poisonous gases as their byproducts. Other methods used in producing functionally graded coating include: plasma spraying, electrodeposition, electrophoretic, Ion Beam Assisted Deposition (IBAD), Self-Propagating High-temperature Synthesis (SHS).

Powder metallurgy (PM) technique is used to produce functionally graded material through three basic steps namely: weighing and mixing of powder according to the pre-designed spatial distribution as dictated by the functional requirement, stacking and ramming of the premixed-powders, and finally sintering. PM technique gives rise to a stepwise structure. If continuous structure is desired, then centrifugal method is used.

Figure 1.2 Powder Metallurgy Process

Centrifugal method is similar to centrifugal casting where the force of gravity is used through spinning of the mould to form bulk functionally graded material. The graded material is produced in this way because of the difference in material densities and the spinning of the mould. There are other similar processes like centrifugal method in the literature (e.g. gravity method, etc.). Although continuous grading can be achieved using centrifugal method but only cylindrical shapes can be formed. Another problem of centrifugal method is that there is limit to which type of gradient can be produced because the gradient is formed through natural process (centrifugal force and density difference). To solve these problems, researchers are using alternative manufacturing method known as solid freeform.

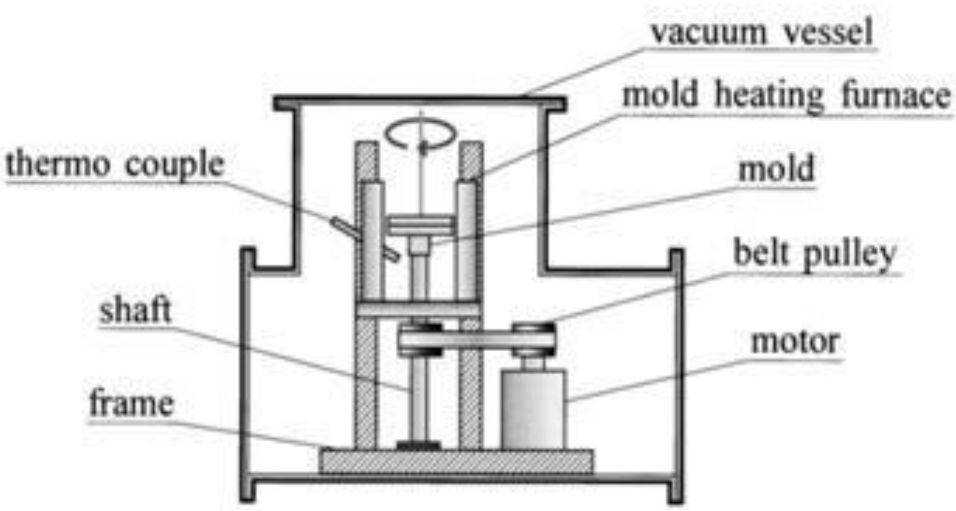

Figure 1.3 Centrifugal Method

Solid freeform is an additive manufacturing process that offers lots of advantages that include: higher speed of production, less energy intensive, maximum material utilization, ability to produce complex shapes and design freedom as parts are produced directly from CAD (e.g. AutoCAD) data. SFF involves five basic steps: generation of CAD data from the software like AutoCAD, Solid edge, conversion of the CAD data to Standard Triangulation Language (STL) file, slicing of the STL into two dimensional cross-section profiles, building of the component layer by layer, and lastly removal and finishing. There are various types of SFF technologies, laser based processes are mostly employed in fabrication of functionally graded materials. Laser based SFF process for FGM include: laser cladding based method, Selective Laser Sintering (SLS), 3-D Printing (3-DP), and Selective Laser Melting (SLM). Laser cladding based system and selective laser melting are capable of producing fully dense components. Solid freeform provide manufacturing flexibility amongst other advantages but the technology is characterized by poor surface finish making it necessary to carry out a secondary finishing operation. There are lots of research efforts in this direction to improve surface finish, dimensional accuracy etc. There are other fabrication methods for functionally graded materials; readers can refer to the review studies by Kieback and Neubrand; and Gasik. These authors presented comprehensive processing techniques of functionally graded materials.

4. TYPES OF FGM

FGM are classified according to different criteria like,

(1) According to the structure, two types, one is continuously structured FGM and another is discontinues (Layard) FGM. Difference is clarified by the figure 1.4-(a) and figure 1.4-(b).

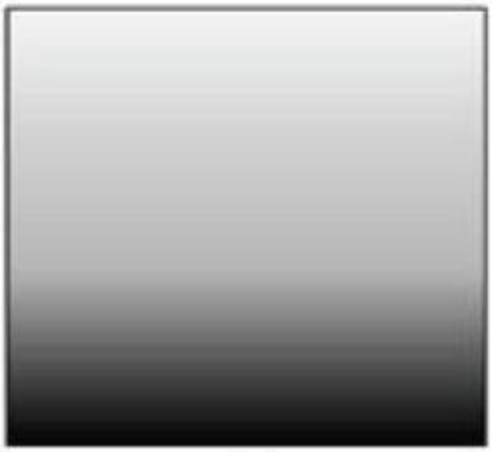

Figure 1.4-(a)

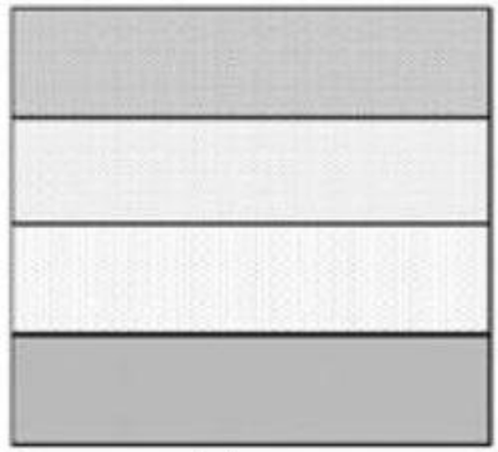

Figure 1.4-(b)

(2) According to process of manufacturing, one type is Thin FGM and another type is Bulk FGM.

Thin FGM are manufactured with different methods like Physical Vapor Deposition (PVD), Chemical Vapor Deposition (CVD), Self-propagating High temperature Synthesis (SHS) method etc. While the Bulk FGMs are manufactured by the methods like, Powder Metallurgical Technique, Centrifugal Casting method, Solid Free Foam technique, etc. Here we concentrate on bulk FGM only, so three main manufacturing techniques are described briefly. In powder Metallurgy method for making bulk FGM, basic steps are weighing and mixing of powder, stacking and ramming of premixed powder, and finally sintering

5 APPLICATIONS OF FGMs

Some of the applications of functionally graded materials are highlighted below, shown in Figure 1.5.

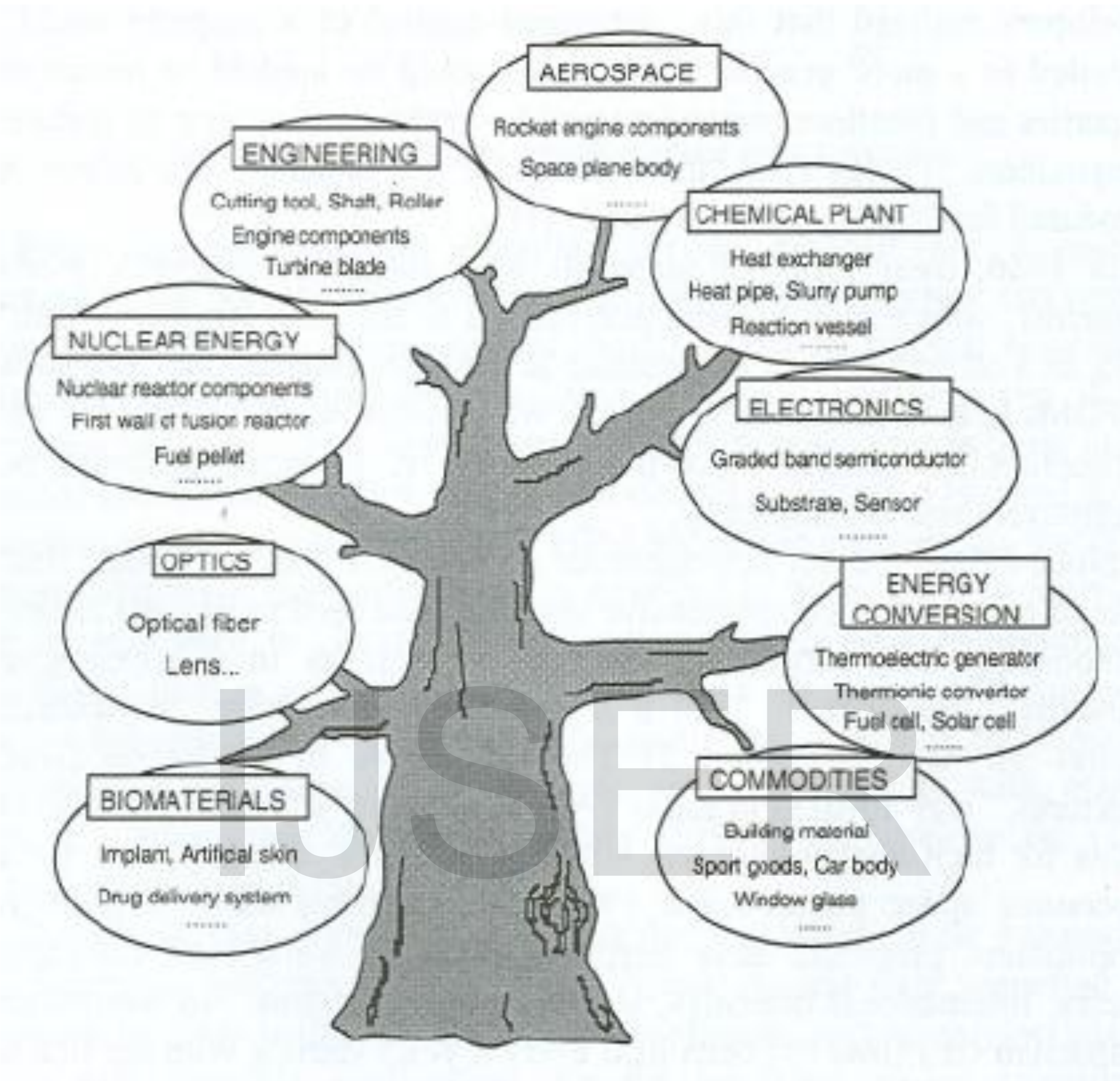

Figure 1.5 Applications of FGMs

A. Aerospace

Functionally graded materials can withstand very high thermal gradient, this makes it suitable for use in structures and space plane body, rocket engine component etc. If processing technique is improved, FGM are promising and can be used in wider areas of aerospace.

B. Medicine

Living tissues like bones and teeth are characterized as functionally graded material from nature, to replace these tissues, a compatible material is needed that will serve the purpose of the original bio-tissue. The ideal candidate for this application is functionally graded material. FGM has find wide range of application in dental and orthopedic applications for teeth and bone replacement.

C. Defense

One of the most important characteristics of functionally graded material is the ability to inhibit crack propagation. This property makes it useful in defense application, as a penetration resistant materials used for armour plates and bullet-proof vests.

D. Energy

FGM are used in energy conversion devices. They also provide thermal barrier and are used as protective coating on turbine blades in gas turbine engine.

E. Optoelectronics

FGM also finds its application in optoelectronics as graded refractive index materials and in audio-video discs magnetic storage media. Other areas of application are: cutting tool insert coating, automobile engine components, nuclear reactor components, turbine blade, heat exchanger, Tribology, sensors, fire retardant doors, etc. The list is endless and more application is springing up as the processing technology, cost of production and properties of FMG improve.

6 ADVANTAGES OF FGMs

The following noticeable advantages are as follows:

i. Provide multi-functionality
ii. Provide ability to control deformation, dynamic response, wear, corrosion, etc. and ability to design for different complex environments
iii. Provide ability to remove stress concentrations
iv. Provide opportunities to take the benefits (pros) of different material systems [e.g. ceramics and metals such as resistance to oxidation

(rust), toughness, machinability, and bonding capability]

7. CHALLENGES OF FGMs

The main challenges and drawbacks of FGMs are:

i. Repeatability of generation procedures and unwavering quality i.e. reliability of the created FGMs.
ii. Conversion of manufacturing process into mass production.
iii. Cost control of production processes.
iv. Quality control of production processes.

8. FORMULATION

Functionally graded beam with top surface as aluminum and bottom surface as zirconia is shown in figure 1.6. The beam, fixed at one end and other end is free as cantilever beam is subjected to a normal load at the free end of the cantilever beam.

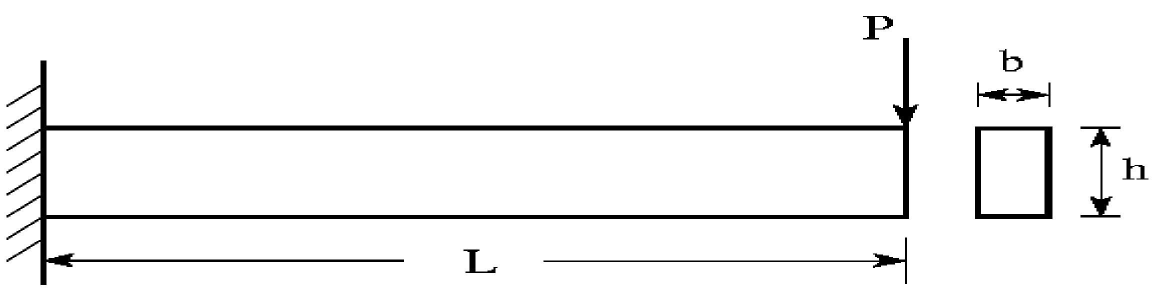

Figure 1.6 functionally graded beam subjected to normal load

The coordinate system of beam element is shown in figure 5.2. The x-y plane chosen as the reference plane for expressing the displacements as shown in figure 1.7. The thickness coordinate is measured as z from the reference plane.

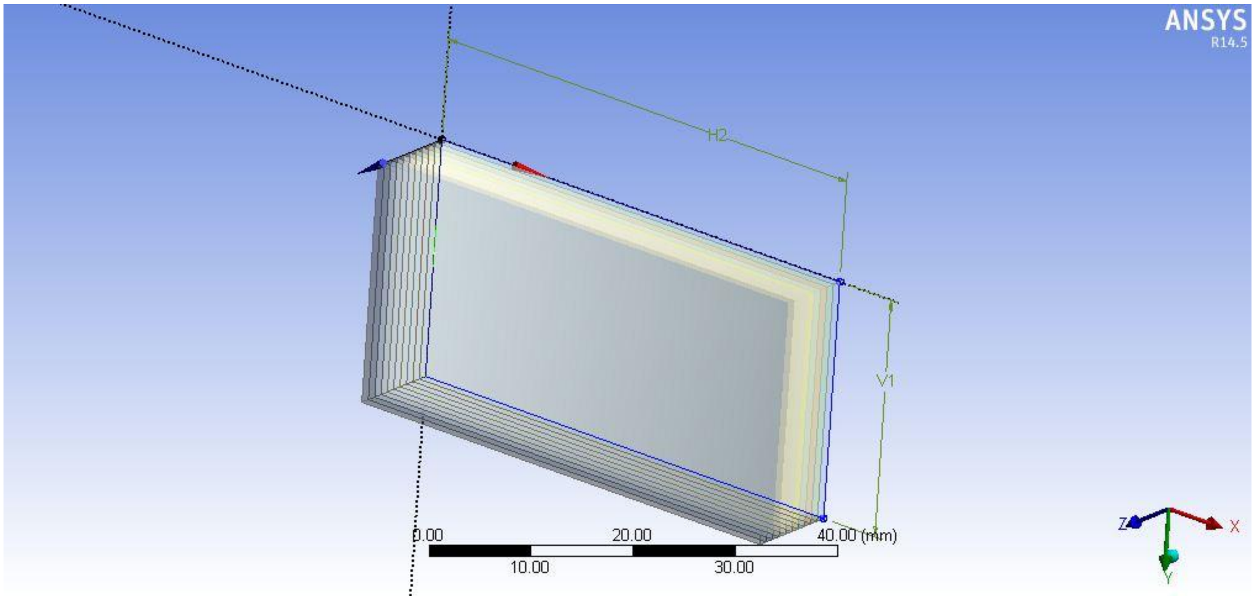

Figure 1.7The coordinate system for FG beam element

Here we assume that the material property gradation is through the thickness and we represent the profile for volume fraction variation by the expression of power law, i.e.

$$P(z) = (Pt - Pb)V + Pb$$

$$V = \left(\frac{z}{h}\right)^n$$

For the material index (n)=2;

At bottom layer, (z/h)=0 and so V-=0

hence P(z) = Pb

At top layer , (z/h)=1 and so V=1

hence P(z)= Pt

International Journal of Scientific & Engineering Research Volume 8, Issue 7, July-2017
ISSN 2229-5518

Where, P denotes a generic material property like modulus, Pt and Pb denote the property of the top and bottom faces of the plate, respectively, h is the total thickness of the plate, and n is a parameter that dictates the material variation profile through the thickness.

9.GOVERNING EQUATION

In order to compute the value of bending stresses developed in a loaded beam, let us consider the two cross-sections of a beam HE and GF, originally parallel as shown in fig 1.8(a, b).when the beam is to bend it is assumed that these sections remain parallel i.e. H'E' and G'F', the final position of the sections, are still straight lines, they then subtend some angle q. Consider now fiber AB in the material, at a distance y from the N.A, when the beam bends this will stretch to A'B'

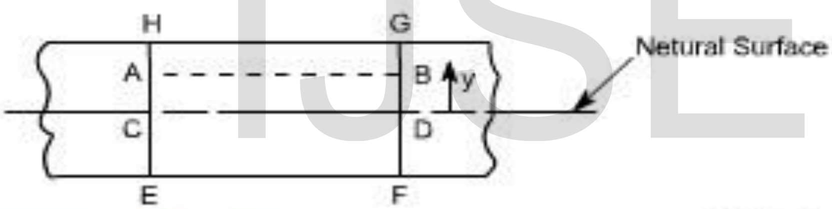

Figure1.8 (a) Before bending,

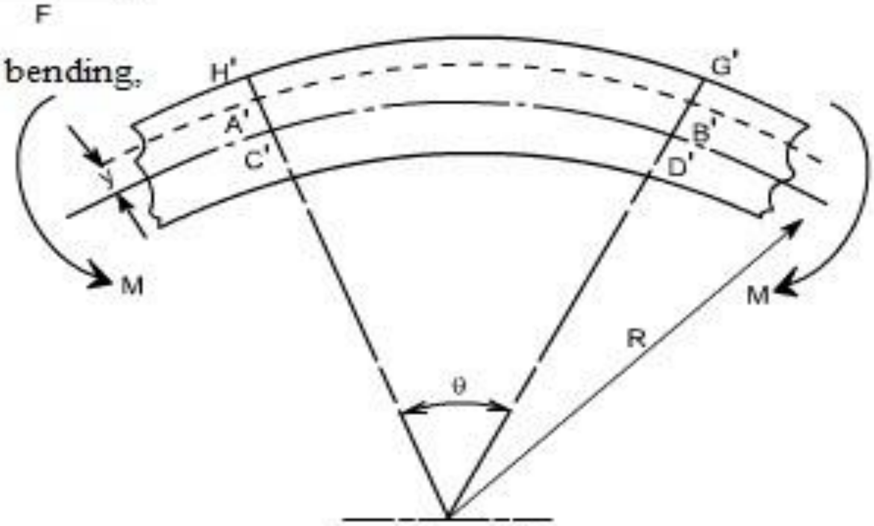

Figure 1.8(b) After Bending

We know that when a beam is under bending the fibers at the top will be lengthened due to tension while at the bottom will be shortened due to compression provided the bending moment M acts at the ends.

$$\frac{\sigma}{y} = \frac{M}{T} = \frac{E}{R}$$

10. Material Properties Modeling

i. Variation of Young’s Modulus (E) along the thickness

The variation of Young’s Modulus, along the thickness of zirconia- aluminum functionally graded rectangular cantilever beam with aluminum rich top surface is shown in Figure **1.9.**

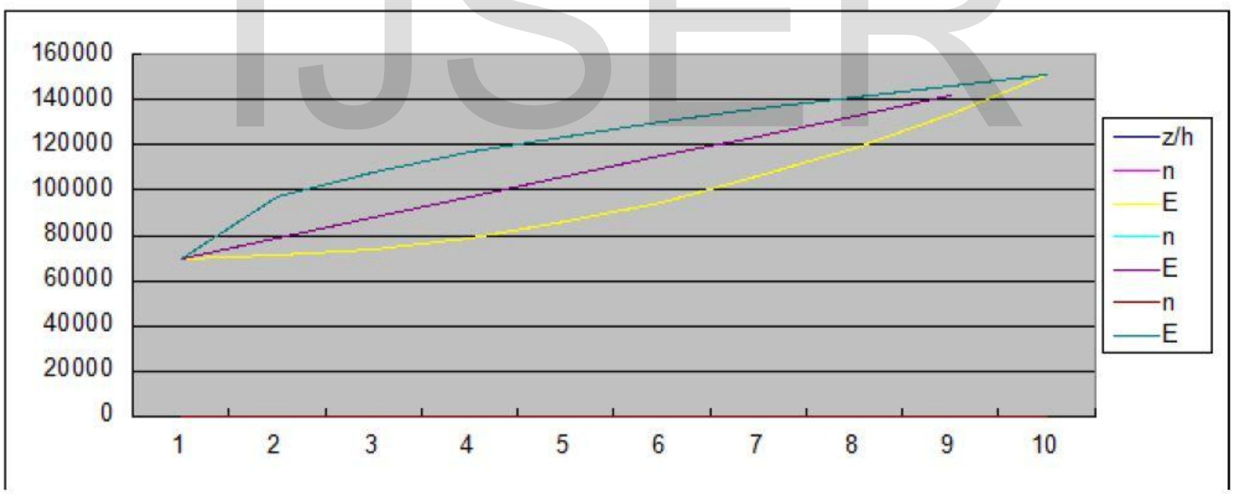

Figure 1.9.The variation of Young’s Modulus, along the thickness

ii. Variation of Density (ρ) along the thickness

The variation of Density, along the thickness of zirconia- aluminum functionally graded rectangular cantilever beam with aluminum rich outer surface is shown in Figure 1.10.

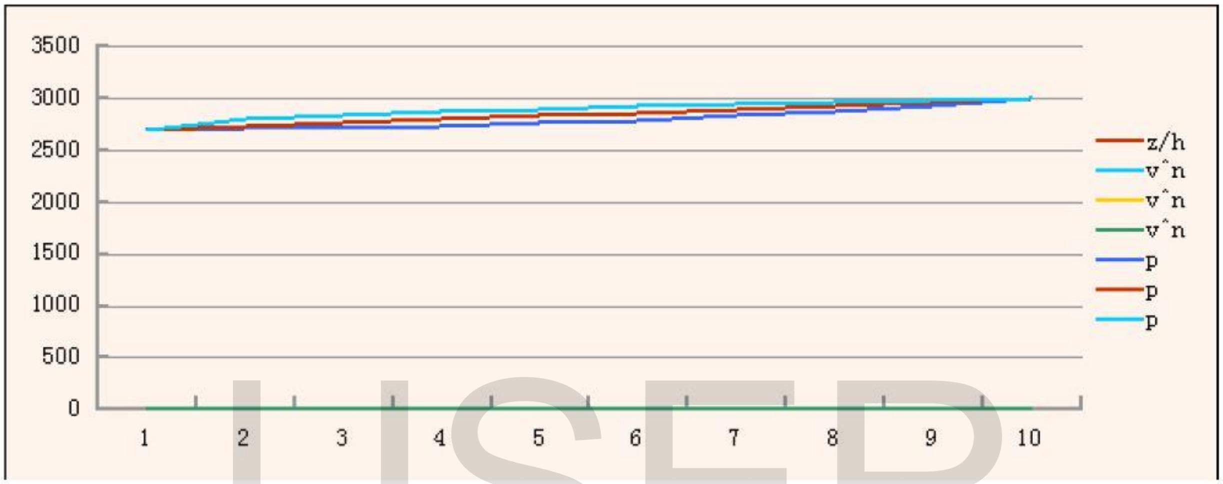

Figure 1.10The variation of Density, along the thickness

11. ANSYS Simulation

11.1Bending Stress by ANSYS

The variation of bending stress of functionally graded rectangular cantilever beam is presented in fig

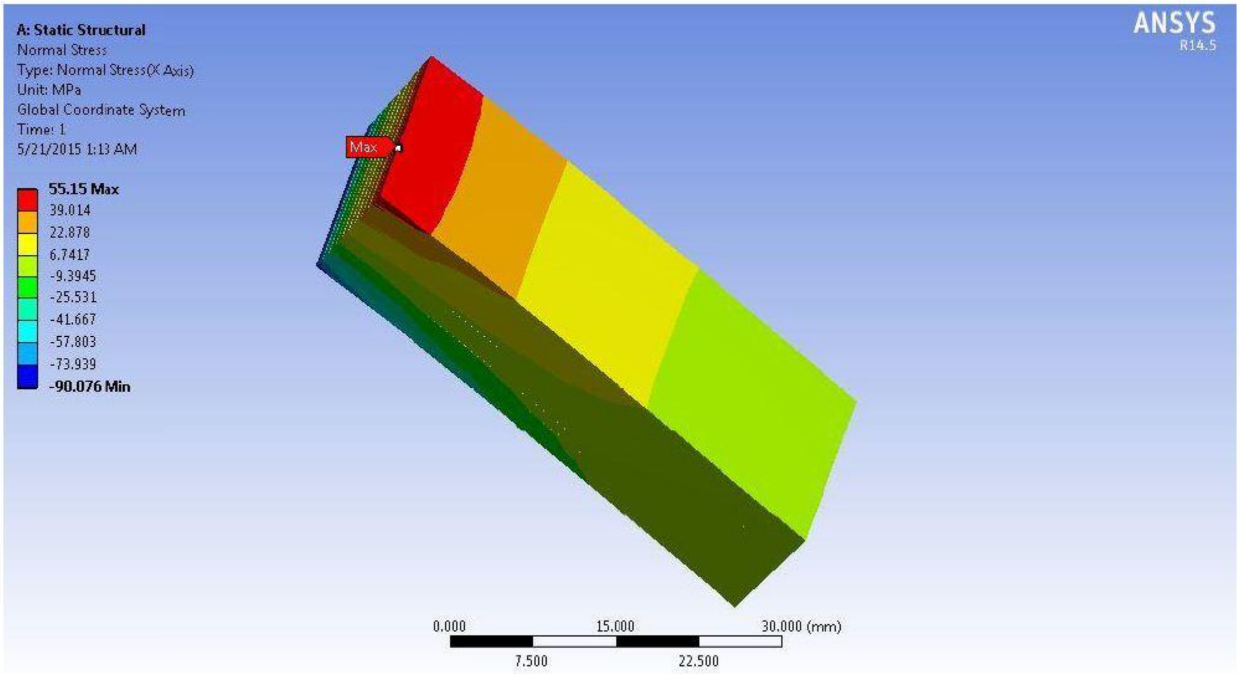

Figure .The variation of bending stress (As can be seen the maximum bending stress concentration is indicated by red and the minimum by blue)

11.2 Bending Strain by ANSYS

The variation of bending strain of functionally graded rectangular cantilever beam is presented in Figure.

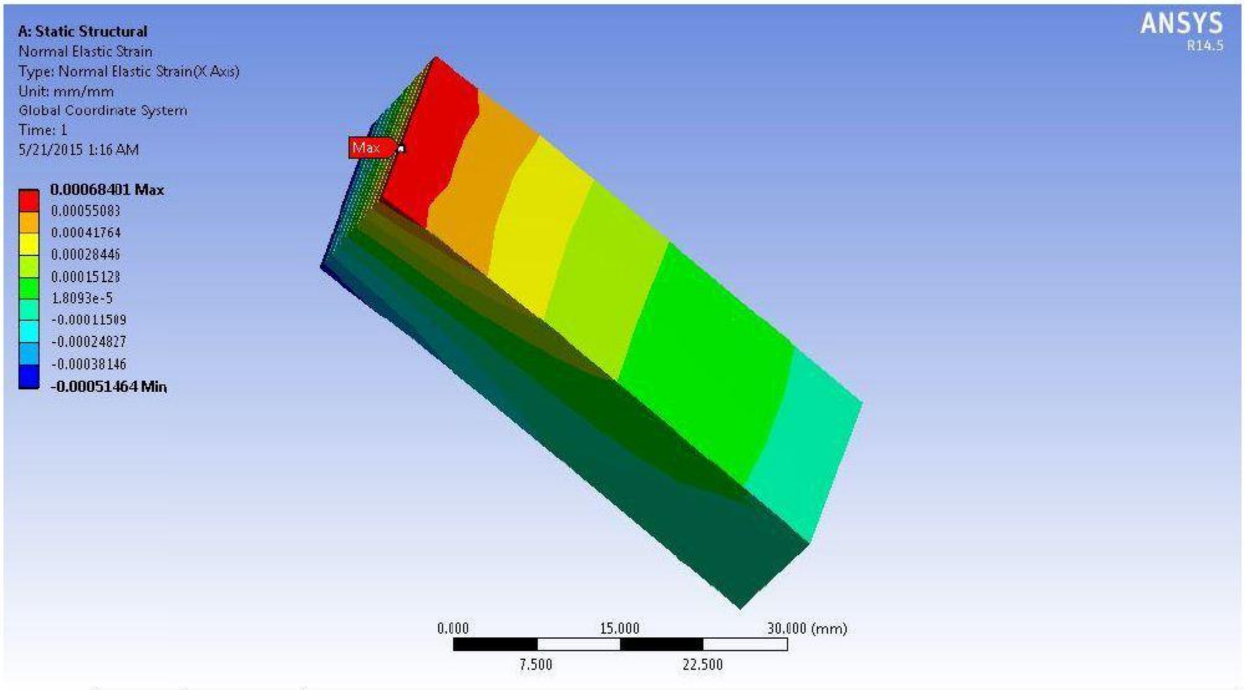

Figure .The variation of bending strain (As can be seen the maximum bending strain concentration is indicated by red and the minimum by blue)

12. COMPARATIVE ANALYSIS:

The dimensions of the rectangular cantilever beam are:
Length L = 0.05m
Breadth = 0.025m and
Thickness = 0.01m.
Normal load P = 1000
The comparison is provided in the table below:

Max Stress(MPa)/ Max Strain	Theoretical Method	ANSYS Workbench 14.5	Differences (%)
Max Bending Stress	48MPa	55.12MPa	12.97
Max Bending Strain	6.857*10^-4	6.84*10^-4	0.3

REFERENCES

1. Rasheedat M. Mahamood, Esther T. Akinlabi, (2012.) "Functionally Graded Material: An Overview. World Congress on Engineering" 2012 Vol III WCE 2012, July 4 - 6, 2012, London, U.K.
2. J.N. Sharma1, Dinkar Sharma2 §, Sheo Kumar3 (2012). "STRESS AND STRAIN ANALYSIS OF ROTATING FGM THERMOELASTIC CIRCULAR DISK BY USING FEM". International Journal of Pure and Applied Mathematics Volume 74 No. 3 2012, 339-352.
3. HASAN, (2010), "Stress analysis in a functionally graded disc under mechanical loads and a steady state temperature Distribution" Vol. 36, Part 1, February 2011, pp. 53–64. Indian Academy of Sciences.
4. By Manoj Sharma, Manish Bhandari & Dr. Kamlesh Purohit, (2013), "Deflection of Functionally Gradient Material Plate under Mechanical, Thermal and Thermo-mechanical Loading", Online ISSN: 2249-4596 & Print ISSN: 0975-586.
5. Sabah Khan, " Analysis of Tribological Applications of Functionally Graded Materials in Mobility Engineering" International Journal of Scientific & Engineering Research Volume 6, Issue 3, pp 1150-1160,March 2015, View at Google Scholar ·View at Scopus
6. Abdur Rosyid, Mahir Es-Saheb and Faycal Ben Yahia, (2014), "Stress Analysis of Nonhomogeneous Rotating Disc with Arbitrarily Variable Thickness Using Finite Element Method" ISSN: 2040-7459; e-ISSN: 2040-7467.
7. Hasan C¸ allio˘glu · Numan Behl¨ ul Bektas¸ · Metin Sayer, (2011),"Stress analysis of functionally graded rotating discs: analytical and numerical solutions", 10409-011-0499-8.

8. Hassan Zafarmand · Behrooz Hassani, (2014),"Analysis of two-dimensional functionally graded rotating thick disks with variable thickness", 10.1007/s00707-013-0966-3.
9. J. Suresh Kumar1, B. Sidda Reddy2,*, C. Eswara Reddy3, K. Vijaya Kumar Reddy4, (2011), "Geometrically non linear analysis of functionally graded material plates using higher order theory", 279-288.
10. Abdur Rosyid, Mahir Es-Saheb and Faycal Ben Yahia, (2014), "Stress Analysis of Nonhomogeneous Rotating Disc with Arbitrarily Variable Thickness Using Finite Element Method", 2040-7467.
11. Ankit B. Makwana Khushbu C. Panchal,(2014), "A Review of Stress Analysis of Functionally Graded Material Plate with Cut-out", 2278-0181.
12. Abdulrahman Soufi Mohammadi, Peyman sabokroohieh2, Ali Falakian3, Amin Hadi , Mohammad Hosseini and Farshad Ehsani, (2013), "Stress and Strain Analysis of Functionally Graded Annular Plate Subjected to Transverse Loading", 2277-2502.
13. Srinivas G, Prasad Shiva U., Manikandan M. and Kumar Praveen A., "Simulation of Traditional Composites under Thermal Loads", *Research Journal of Recent Sciences,* 2(ISC-2012), 273-278 (2013).
14. Sankar B.V., An elasticity solution for functionally graded beams, "*Composites Sciences and Technology"*, 61(5), 689- 696, (2001).
15. Chakraborty A., Gopalakrishnan S. and Reddy J.N., "A new beam finite element for the analysis of functionally graded materials", *International Journal of Mechanical Sciences*, 45(3), 519-539 (2003).